**SYNOPSIS**

**Ph.D in Mathematics**

# Exploring the $\delta$ – Semiclosed Sets in Topological Spaces and Applications in Decision Making Problem


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## Introduction

Topology is the study of shapes and spaces. Topology originated from mathematics in 19th century. This filed developed into well-designed and versatile domain of general topology, which blossoms in both pure and applied mathematics. Now a days this field of research is more actively doing by the researchers. The concept of open sets, which serve as powerful tools for defining the structure of topological spaces. Stone (1937) introduced regular open sets, establishing a connection between Boolean Rings and topology. Later, Velicko (1968) proposed the concept of δ-open sets to generalize regular open sets. These δ-open sets provided deeper insights into H-closed spaces and led to the creation of a new topology $\tau_\delta$, which is stronger than the standard topology τ. Levine (1963) initiated such generalizations by introducing the concepts of semi open sets and generalized open sets in topological spaces. Generalized closed sets (briefly g - closed sets) were introduced in order to extend some important properties of closed sets to a larger family of sets. Park, et.al., (1997) introduced the notion of δ - semi open sets which are stronger than semi open sets but weaker than δ-open sets. Georgiou (2004) developed the theory on generalization of $\delta$ - closed sets which was named as $(\Lambda,\delta)$ - closed sets using Λ– operator in terms of $\delta$.

With these works as the motivation, yet another generalization of δ – semiclosed sets namely $\Lambda_{\delta S}$ sets, $(\Lambda,\delta S)$ – closed sets and $\lambda_g^{\delta S}$ – closed sets are introduced and the same sets are applied in separation axioms, continuous functions and homeomorphisms. These concepts and their properties are extended to Pythagorean Nano Topological Space and Hesitant Fuzzy Soft Topological Space.

## Objective of the research

- To introduce and investigate the generalization of $\delta$- closed sets
- To extend the concept to Pythagorean Nano Topological Space and Hesitant Fuzzy Soft Topological Space
- To introduce new score functions in Pythagorean Nano Topological Space
- To evaluate the performance of the score functions

- To apply score functions to multi attribute decision making problem

**Methodology**

The research has been done by the following methods.

- Analytical method of comparing ($\Lambda$,$\delta$S) – closed sets and $\lambda_g^{\delta S}$- closed sets with existing closed sets
- Producing counter example wherever necessary to substantiate the result
- Analysis of preservation of topological properties by ($\Lambda$,$\delta$S) – closed sets and $\lambda_g^{\delta S}$-closed sets.
- Obtaining characterization theorems
- New score functions on Pythagorean Nano Topological Space are constructed and validated by existing data
- Multi attribute decision making problem is analysed and Pythagorean nano topology is used to solve it.

**Notations**

Throughout the thesis, the following notations are used.

- (P, $\sigma$), (Q, $\tau$) and (R, $\eta$) denote non-empty topological spaces on which no separation axioms are mentioned, unless it is stated specifically.
- In all the diagrams A $\rightarrow$ B represents A implies B but not conversely and A $\nleftrightarrow$ B represents A and B are independent of each other.
- In this Introduction part, Definitions are indicated by ■ and Results are indicated by •.

**Chapter 1**

Chapter 1 deals with preliminary definitions and related concepts on various existing closed sets which are used in the remaining part of the study.

**Chapter 2**

The main aim of this Chapter is to introduce an operator $\Lambda_{\delta s}$(R) using δ-semiopen sets and the concept of ($\Lambda$,δS)-closed sets in topological spaces. The notions $\Lambda_{\delta s}$(R), $\Lambda_{\delta s}$ - sets, $\Lambda_{\delta s}$(R), $\Lambda_{\delta s}$-set, ($\Lambda$,δS)-closed sets are defined and their properties are studied. The concept of separation axioms by δ−semiopen sets and the δ−semiclosure. The notions δ−semi R0,

δ−semi D, δ−semi Di (i = 0, 1, 2) and δ−semi Ti (i = 0, 1, 2) are defined and their properties are studied.

**Important definitions and results:**

- A subset R of a topological space $(P, \sigma)$ is called a $\boldsymbol{\Lambda_{\delta s}}$ **– set** if R = $\Lambda_{\delta s}$(R). The family of all $\Lambda_{\delta s}$ – sets of a topological space (P,$\sigma$) is denoted by $\boldsymbol{\Lambda_{\delta s}(P,\sigma)}$
- A subset R of a topological space (P,$\sigma$) is called $\boldsymbol{(\Lambda, \delta s)}$ **– closed** if R = T ∩ C, where T is a $\Lambda_{\delta s}$ – set and C is a $\delta$ −semiclosed set.
- Every $\Lambda_{\delta s}$ – set (resp.$\delta$ − semiclosed set) is $(\Lambda, \delta s)$ – closed
- A topological space (P, $\sigma$) is a $\delta$ - semi $D_0$ space if and only if for each x ∈ $P$, the singleton {x} is $(\Lambda, \delta s)$ – closed
- A topological space (P,$\sigma$) is $\delta$ − semi $T_1$ if and only if for each x ∈ P, the singleton {x} is a $\Lambda_{\delta s}$ – set
- A subset R of a topological space (P,σ) is called $\Lambda^*_{\delta S}$-set if R = $\Lambda^*_{\delta S}$(R).
- For topological space (P,σ), we set $\sigma^{\Lambda^*_{\delta S}}$= {R | R is a $\Lambda^*_{\delta S}$-set of P}, then the pair (P, $\sigma^{\Lambda^*_{\delta S}}$) is an Alexandroff Space.

**CHAPTER III**

In this Chapter, concerned with some weaker forms of $(\Lambda, \delta S)$−open sets namely s$(\Lambda, \delta S)$−open sets, p$(\Lambda, \delta S)$−open sets, $\alpha(\Lambda, \delta S)$−open sets, and $\beta(\Lambda, \delta S)$−open sets and a stronger form of $(\Lambda, \delta S)$−open sets which is r$(\Lambda, \delta S)$−open sets. Many important properties and characterizations relating these new concepts are obtained and well established. These concepts $\Lambda_{\delta S}$-extremally disconnected spaces, $\Lambda_{\delta S}$-sub maximal spaces and $\Lambda_{\delta S}$-hyperconnected spaces are introduced here. The main aim of this article is to establish some important characterizations of $\Lambda_{\delta S}$-extremally disconnected spaces, $\Lambda_{\delta S}$ – submaximal spaces and $\Lambda_{\delta S}$ -hyper connected spaces.

**Important definitions and results:**

- A subset R of a topological space (P,$\sigma$) is said to be:

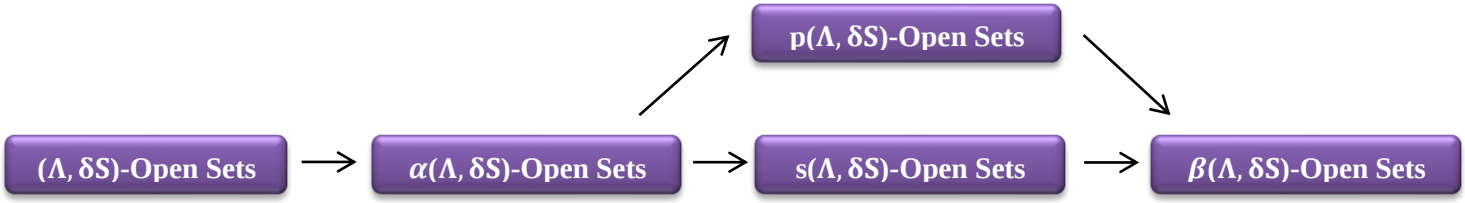
a. $\mathbf{s(\Lambda, \delta S)}$**-open** if R ⊆ $\Lambda_{\delta S}\mathrm{Cl}(\Lambda_{\delta S}\mathrm{Int}(\mathrm{R}))$

b. $\mathbf{p(\Lambda, \delta S)}$**-open** if R ⊆ $\Lambda_{\delta S}\mathrm{Int}(\Lambda_{\delta S}\mathrm{Cl}(\mathrm{R}))$

c. $\boldsymbol{\alpha(\Lambda, \delta S)}$**-open** if $\mathrm{R} \subseteq \Lambda_{\delta S}\mathrm{Int}\left(\Lambda_{\delta S}\mathrm{Cl}\left(\Lambda_{\delta S}\mathrm{Int}(\mathrm{R})\right)\right)$

d. $\boldsymbol{\beta(\Lambda, \delta S)}$**-open** if $\mathrm{R} \subseteq \Lambda_{\delta S}\mathrm{Cl}\left(\Lambda_{\delta S}\mathrm{Int}\left(\Lambda_{\delta S}\mathrm{Cl}(\mathrm{R})\right)\right)$

**The following diagram shows the relation of $(\Lambda,\delta S)$ – Open Sets with other $(\Lambda,\delta S)$ – Open Sets in topological space**



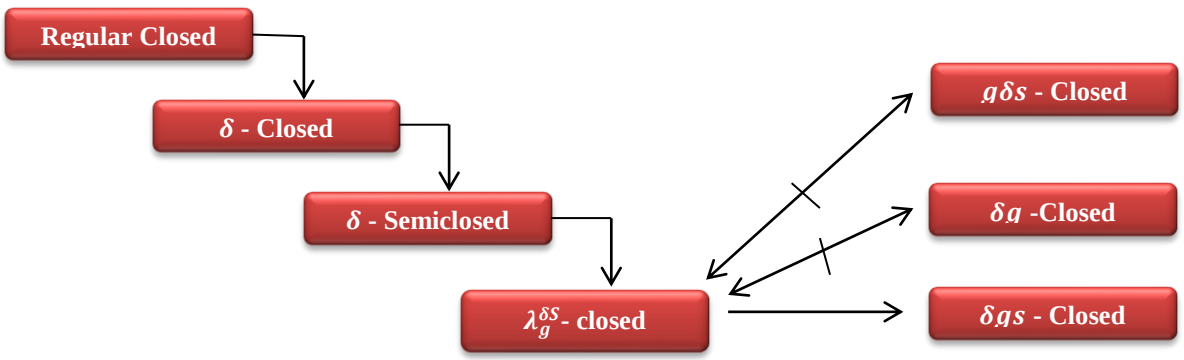

## Chapter IV

The aim of this chapter is to deals with yet another notion namely $\lambda_g^{\delta S}$-closed sets and their properties. Meanwhile, $\lambda_g^{\delta s}$-closed sets stand isolated from δg – closed, $\lambda_g^{\delta}$-closed and gδs – closed sets. Arbitrary union of $\lambda_g^{\delta S}$-closed sets is $\lambda_g^{\delta S}$-closed, whereas finite intersection of them fails. This is supported by a suitable counter example. The notions of $\lambda_g^{\delta S}$-open set, $\lambda_g^{\delta S}$-closure operator, $\lambda_g^{\delta S}$-interior operator are developed and their properties are analyzed.

**Important definitions and results:**

- A subset R of a topological space (P,$\sigma$) is called **$\lambda_g^{\delta S}$- closed sets** if $sCl_{\delta}$(R) ⊆ U whenever R ⊆ U and U is $(\Lambda, \delta S)$-open in P.

**This diagram represents dependent and independent relationship between $\lambda_g^{\delta S}$- closed sets with other existing closed sets**



**Chapter V**

Chapter 5, is to introduce a new class of neighborhoods using $\lambda_g^{\delta S}$-open sets namely $\lambda_g^{\delta S}$-neighborhoods in topological spaces. Further, we discuss some vital relations and interesting characterizations of $\lambda_g^{\delta S}$-neighborhood. This new notion can be applied in the Geographic Information System (GIS), where the concept of nearness is studied in terms of the relation between objects rather than the distance between them. In a consequent manner, concepts of $\lambda_g^{\delta S}$-Frontier, $\lambda_g^{\delta S}$-Boundary, $\lambda_g^{\delta S}$-Exterior and $\lambda_g^{\delta S}$-Saturated set are studied. $\lambda_g^{\delta S}$-open sets are analysed through grills.

**Important definitions and results:**

- A subset N of a topological space (P,$\sigma$) is called a $\boldsymbol{\lambda_g^{\delta S}}$**-neighborhood of p** $\in$ **P** if there exists a $\lambda_g^{\delta S}$-open set Q such that p $\in$ Q $\subseteq$ N.
- A subset N of R $\subseteq$ P is called a $\boldsymbol{\lambda_g^{\delta S}}$**- neighborhood of R** if there exists a $\lambda_g^{\delta S}$- open set A such that N $\subseteq$ A $\subseteq$ R.
- If a subset N of a topological space P is $\lambda_g^{\delta S}$- open then N is a $\lambda_g^{\delta S}$- neighborhood of each of its points.
- Let p $\in$ R $\subseteq$ P. Then p is called a $\boldsymbol{\lambda_g^{\delta S}}$**-limit point** of R if every $\lambda_g^{\delta S}$-open set containing p contains at least one point of R other than p. That is, N $\cap$ (R \ {p}) $\neq \emptyset$, for all N $\in \lambda_g^{\delta S}$O(p). The set of all $\lambda_g^{\delta S}$-limit points of R $\subseteq$ P is called the $\boldsymbol{\lambda_g^{\delta S}}$**-derived set** of R and denoted by $\lambda_g^{\delta S}$D(R).
- For a subset R of a topological space (P,$\sigma$), $\boldsymbol{\lambda_g^{\delta S}}$**-frontier of R** is denoted by $\boldsymbol{\lambda_g^{\delta S}}$**F (R)** and defined as $\lambda_g^{\delta S}$F (R) = $\lambda_g^{\delta S}Cl$(R) \ $\lambda_g^{\delta S}Int(R)$.
- For a subset R of a topological space (P,$\sigma$), $\boldsymbol{\lambda_g^{\delta S}}$**-boundary of R** is denoted by $\boldsymbol{\lambda_g^{\delta S}}$**B(R)** and defined as $\lambda_g^{\delta S}$B(R) = R \ $\lambda_g^{\delta S}$Int(R).
- For a subset R of a topological space **(P,$\sigma$),** $\boldsymbol{\lambda_g^{\delta S}}$**-exterior** of R is denoted by $\boldsymbol{\lambda_g^{\delta S}}$**E(R)** and defined as $\lambda_g^{\delta S}$E(R) = P \ $\lambda_g^{\delta S}$Cl(R).
- A subset R of a topological space (P,$\sigma$) is said to be $\boldsymbol{\lambda_g^{\delta S}}$**-saturated** if $\lambda_g^{\delta S}$Cl({x}) $\subseteq$ R for every x $\in$ R.

**Chapter VI**

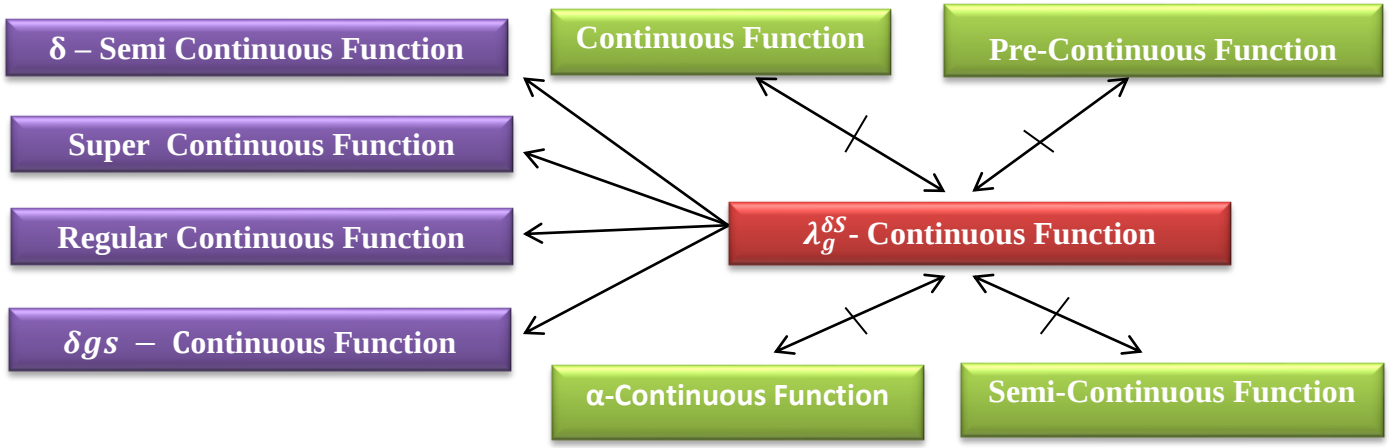
In this chapter ($\Lambda$,$\delta$S) continuity using ($\Lambda$,$\delta$S) – open sets and some of their properties are analysed. $\lambda_g^{\delta S}$ continuity (resp., Quasi $\lambda_g^{\delta S}$ continuity, Perfectly $\lambda_g^{\delta S}$ continuity,

Totally $\lambda_g^{\delta S}$ continuity, Strongly $\lambda_g^{\delta S}$ continuity, contra $\lambda_g^{\delta S}$ continuity) are also defined using $\lambda_g^{\delta S}$ - open sets and we extend various results and properties to these concerned concepts.

**Important definitions and results:**

- A map f : (P,$\sigma$) → (Q,$\tau$) is called $(\boldsymbol{\Lambda, \delta S})$ - **continuous function** if the inverse image $f^{-1}$(V) of each open set V in (Q,$\tau$) is $(\Lambda, \delta S)$ –open in (P,$\sigma$)
- A function f : (P, $\sigma$) → (Q, $\tau$) is said to be $\lambda_g^{\delta S}$-continuous function if and only if inverse image of a closed set in (Q, $\tau$) $\lambda_g^{\delta S}$-closed in (P, $\sigma$)

**Dependent and Independent relationships**




**Chapter VII**

In this chapter, we delve into the concepts of $\lambda_g^{\delta S}$- open and $\lambda_g^{\delta S}$- closed functions, examining their behaviors within the context of almost weakly Hausdorff spaces. We also introduce two distinct types of homeomorphisms: $\lambda_g^{\delta S}$- homeomorphisms and $\lambda_g^{\delta S^*}$- homeomorphisms, and analyze their respective properties. Notably, it is observed that the collection of all $\lambda_g^{\delta S^*}$- homeomorphisms forms a group when composed together.

**Important definitions and results**

- A function f : (P,$\sigma$ ) → (Q, $\tau$) is

a. $\boldsymbol{\lambda_g^{\delta S}}$ **– open** if f(R) is $\lambda_g^{\delta S}$ – open in (Q, $\tau$) for every open set R in (P, $\sigma$).

b. $\boldsymbol{\lambda_g^{\delta S}}$ **– closed** if f(R) is $\lambda_g^{\delta S}$ – closed in (Q, $\tau$) for every closed set R in (P, $\sigma$).

- For topological spaces (P, $\sigma$), (Q, $\tau$), (R, $\eta$), f : (P, $\sigma$) → (Q, $\tau$), g : (Q, $\tau$) → (R, $\eta$) and $g \circ f$ : (P, $\sigma$) → (R, $\eta$) the following results are true.

a. If $g \circ f$ is $\lambda_g^{\delta S}$-open and f is continuous, surjective then g is $\lambda_g^{\delta S}$-open

b. If $g \circ f$ is open and g is $\lambda_g^{\delta S}$-continuous, injective then f is $\lambda_g^{\delta S}$-open

- A bijective function f : (P, σ) → (Q, τ) is called **$\lambda_g^{\delta S}$-homeomorphism** if f is both $\lambda_g^{\delta S}$-continuous and $\lambda_g^{\delta S}$-open.
- . A bijective function f : (P, σ) → (Q, τ) is called **$\lambda_g^{\delta S^*}$- homeomorphism** if both f and $f^{-1}$ are $\lambda_g^{\delta S}$-irresolute.
- The set of all $\lambda_g^{\delta S^*}$-homeomorphisms of (P, σ) onto itself denoted by $\lambda_g^{\delta S^*}h(P, \sigma)$ forms a group under composition of functions.

**Chapter VIII**

In this chapter, The study carried out for $\Lambda_{\delta S}$ – set, $(\Lambda, \delta S)$ – closed sets and $\lambda_g^{\delta S}$ – closed sets in topological spaces are extended to HFSTS. HFS$\Lambda_{\delta S}$ operator. HFS$\Lambda_{\delta S}$-set, HFS $(\Lambda, \delta S)$ – closed sets and HFS $\lambda_g^{\delta S}$ – closed sets are defined initially and their properties in HFSTS are stated.

**Important definitions and results:**

- Let $(hfs, E)$ be a HFS-set of a ${}_{HFS}TS$ (P,$\tilde{\sigma}$,E) . Then ${}_{\boldsymbol{HFS}}\boldsymbol{\Lambda_{\delta s}}(\boldsymbol{hfs}, \boldsymbol{E})$ is defined as follows:
- A HFS - set $(hfs, T)$ of a ${}_{HFS}TS$ (P,$\tilde{\sigma}$,E) is known as a ${}_{\boldsymbol{HFS}}\boldsymbol{\Lambda_{\delta s}}$**– set** if $(hfs, T) = {}_{HFS}\Lambda_{\delta s}(hfs, T)$.
- For a ${}_{\text{HFS}}\delta S(\mathfrak{U}, \sigma_{\Re}(X)))$, Let us denote ${}_{HFS}\Lambda_{\delta s}O(\mathfrak{U}, \sigma_{\Re}(X)) = \{F/F$ is a ${}_{HFS}\Lambda_{\delta s}$-set of P$\}$. Then the pair (P, ${}_{HFS}\Lambda_{\delta s}O(\mathfrak{U}, \sigma_{\Re}(X)))$ is an Alexandroff space.
- A subset T of a ${}_{HFS}TS$ $(\mathfrak{U}, \sigma_{\Re}(X)))$ is called ${}_{\boldsymbol{HFS}}(\boldsymbol{\Lambda}, \boldsymbol{\delta S})$**– closed** if T = S ∩ C, where S is a ${}_{HFS}\Lambda_{\delta s}$ – set and C is a ${}_{\text{HFS}}\delta S -$ closed set. Let T be a subset of ${}_{HFS}TS$ $(\mathfrak{U}, \sigma_{\Re}(X))$. A set ${}_{\boldsymbol{HFS}}\boldsymbol{\Lambda}^*_{\boldsymbol{\delta S}}(\mathbf{T})$ is delineated subsequently: ${}_{HFS}\Lambda^*_{\delta S}(T) = \vee \{S \in {}_{\text{HFS}}\delta SC(U,\sigma_{\Re}(X)) / S \subseteq T\}$.
- A subset T of a ${}_{HFS}TS$ $(\mathfrak{U}, \sigma_{\Re}(X))$ is called ${}_{\boldsymbol{HFS}}\boldsymbol{\Lambda}^*_{\boldsymbol{\delta S}}$ **- Set** if T = ${}_{HFS}\Lambda^*_{\delta S}(T)$.
- For ${}_{HFS}TS$ $(\mathfrak{U},\sigma_{\Re}(X))$, we set $PN\sigma^{\Lambda^*_{\delta s}} = \{T / T$ is a ${}_{HFS}\Lambda^*_{\delta S}$- set of P$\}$, then the pair $(\mathfrak{U}, PN\sigma^{\Lambda^*_{\delta s}})$ is an Alexandroff Space.
- A subset T of a ${}_{HFS}TS$ $(\mathfrak{U},\sigma_{\Re}(X))$ is called ${}_{\boldsymbol{HFS}}\boldsymbol{\lambda_g^{\delta S}}$**-closed sets** if ${}_{\text{HFS}}\delta S_{Cl}(T) \subseteq S$ whenever T ⊆ S and S is ${}_{HFS}(\Lambda, \delta S)$ – open in P.

**Chapter IX**

First is to extend the definitions of $\Lambda_{\delta S}$-sets, ($\Lambda$,$\delta$S) – closed sets and $\lambda_g^{\delta S}$-closed sets to Pythagorean nano topological spaces. Next is to study the properties of these sets in the Pythagorean nano topological framework. Finally, we want to propose more theoretical concepts and show the application of the work in some real-life situations by combining it with wide-area decision–making. There are many existing models which are still developing in this particular area but when we deal with more fuzzified data, this method is more useful without reducing the constraint when compared to the other concepts. The proposed concepts and model have more fuzzified values as information but still hold the same condition as the other models, which have a great advantage in dealing with the more vague details of the problem.

**Important definitions and results:**

- Let T be a subset of a PNTS ($\mathfrak{U}$,$\sigma_{\Re}$(X)). Then **PN$\Lambda_{\delta s}$(T)** is defined as follows:

$$PN\Lambda_{\delta S}(T) = \wedge\{F \in PN\delta SO(\mathfrak{U}, \sigma_{\Re}(X)) / T \subseteq F\}.$$

- A subset T of a PNTS $(\mathfrak{U}, \sigma_{\Re}(X))$ is known as a **PN$\Lambda_{\delta s}$ – set** if T = PN$\Lambda_{\delta s}$(T).
- For a PNTS$(\mathfrak{U}, \sigma_{\Re}(X)))$ , Let us denote PN$\Lambda_{\delta s}$O(U, $\sigma_{\Re}$(X)) = {F/F is a PN$\Lambda_{\delta s}$-set of P}. Then the pair (P, PN$\Lambda_{\delta s}$O($\mathfrak{U}, \sigma_{\Re}(X)$)) is an Alexandroff space.
- A subset T of a PNTS ($\mathfrak{U}, \sigma_{\Re}(X)$)) is called **PN($\Lambda$, $\delta$s) – closed** if T = S ∩ C, where S is a PN$\Lambda_{\delta s}$ – set and C is a PN$\delta$S – closed set.
- Let T be a subset of PNTS ($\mathfrak{U}$, $\sigma_{\Re}$(X)). A set **PN$\Lambda^*_{\delta S}$(T)** is delineated in the subsequent manner:

$$PN\Lambda^*_{\delta S}(T) = \vee \{S \in PN\delta SC(U,\sigma_{\Re}(X)) / S \subseteq T\}.$$

- A subset T of a PNTS ($\mathfrak{U}$, $\sigma_{\Re}(X)$) is called **PN$\Lambda^*_{\delta S}$ - Set** if T = PN$\Lambda^*_{\delta S}(T)$.
- For PNTS ($\mathfrak{U}$,$\sigma_{\Re}(X)$), we set $PN\sigma^{\Lambda^*_{\delta S}}$ = {T / T is a PN$\Lambda^*_{\delta S}$ - set of P}, then the pair ($\mathfrak{U}$,$PN\sigma^{\Lambda^*_{\delta S}}$) is an Alexandroff Space.
- A subset T of a PNTS ($\mathfrak{U}$,$\sigma_{\Re}(X)$) is called $\boldsymbol{PN\lambda_g^{\delta S}}$**-closed sets** if $PN\delta S_{Cl}$(T) ⊆ S whenever T ⊆ S and S is $PN(\Lambda, \delta S)$ – open in P.

## Summary and Conclusion

In this thesis, the study of the notion $\lambda_g^{\delta S}$- closed sets is presented. This notion is properly placed between δ-semi closed sets and δgs-closed sets. It is the nearest weaker form of δ-semiclosed sets. The reverse implications of the dependence relationship, which do not hold good, are substantiated by counter examples. The characterizations of $\lambda_g^{\delta S}$-closed sets in semi-regular space, almost weakly Hausdorff space, T3/4-spaces are also derived.

Properties of various notions related to $\lambda_g^{\delta S}$-closed sets namely $\lambda_g^{\delta S}$-neighborhood, $\lambda_g^{\delta S}$-limit point, $\lambda_g^{\delta S}$-derived set, $\lambda_g^{\delta S}$-frontier, $\lambda_g^{\delta S}$-boundary, $\lambda_g^{\delta S}$-exterior and $\lambda_g^{\delta S}$-saturated set are studied. $\lambda_g^{\delta S}$-open sets are characterized using the concept of grill in topological spaces.

As an application of $\lambda_g^{\delta S}$-closed sets, two new spaces (i.e.,) hesitant fuzzy soft topological space and Pythagorean nano topological spaces are constructed and their interrelations and properties are analyzed.

It is shown that the composition of two $\lambda_g^{\delta S}$-continuous functions is not preserved. But after changing the conditions suitably, the composition is preserved. Characteristics of $\lambda_g^{\delta S}$-irresoluteness and contra $\lambda_g^{\delta S}$- continuous functions are presented.

An application in MADM using PNTS has been proposed and illustrated using a real time example.

The collection of various open sets and closed sets for the topological spaces of three elements and four elements which are tabulated in Appendix I and Appendix II, are constructed to provide counter example whereever necessary.

**References:**


1. Ajay, D. and Charisma, J. J., Pythagorean nano topological space, International Journal of Recent Technology and Engineering, vol. 8, pp. 3415-3419, 2020.
2. Ajay, D. and Charisma, J. J., On week forms of Pythagorean nano open sets, Advances in Mathematics: Scientific Journal, vol. 9, pp.5953-5963, 2020.
3. Ajay, D. and Charisma, J. J. Pythagorean nano continuity, Advances in Mathematics: Scientific Journal, vol. 9, pp.6291-6298, 2020.
4. Ajay, D. Joseline Charisma, J., Petaratip, T., Hammachukiattikul, P., Boonsatit, N., Jihad Younis A., and Rajchakit, G., Pythagorean Nano generalized Closed sets with Application in decision-Making, Journal of Function Spaces, 2021. https://doi.org/10.1155/2021/2571301.
5. Babithaa, K. V. and Sunil Jacob John, Hesitant fuzzy soft sets, Journal of New Results in Science 3 (2013), 98-107
6. Caldas, M., Georgiou, D. N., Jafari, S., Noiri, T., (2003), More on $\delta$-closed sets, Note di Matematica, 22(2).
7. Caldas, M., Georgiou, D. N. and Jafari, S. (2007), Study on (Λ, α)-Closed Sets and the Related Notions in Topological Spaces, Bull. Malays. Math. Sci. Soc. (2), 30(1), 23-36.
8. Dontchev, J. (1995), On submaximal spaces, Tamkang. J. Math., 26, 253-260.
9. Dontchev, J. and Ganster, M. (1996), On δ-generalized closed sets and T3/4-spaces, Mem. Fac. Sci. Kochi Univ. Ser. Math.,17, 15-31.
10. Dontchev, J., Arokiarani, I., Balachandran, K. (2000), On generalized δ-closed sets and almost weakly Hausdorff spaces, Q and A in General Topology,18, 17-30.
11. Dubois, D., Prade, H., Fuzzy sets and systems: theory and applica-tions. Academic Press, New York, 1980.
12. Georgiou, D. N., Jafari, S. and Noiri, T. (2004), Properties of (Λ, δ)-closed sets in topological spaces, Bollettino dell'Unione Matematica Italiana, Serie 8, 7-B, 745-756.
13. Jafari, S. and Noiri, T. (2002), On contra precontinuous functions, Bull. of Malaysian Mathematical Sci. Soc., 25, 115-128.
14. Jin Han Park, Dae Seob Song and Bu Young Lee (2007), On δgs-closed sets and almost weakly Hausdorff Spaces, Honam Mathematical Journal, 29, 597-615.
15. Jin Han Park, Dae Seob and Saadati, R. (2007), On generalized δ-semiclosed sets in topological spaces, Chaos, Soliton & Fractals, 33, 1329-1338.
16. M. Lellis Thivagar, C. Richard "On Nano Continuity", Mathematical Theory and Modelling,vol. 3(7), pp. 32-7, 2013.
17. M. Lellis Thivagar, S. Jafari, V. Sudha Devi and V. Antonysamy, "A novel approach to nano topology via neutrosophic sets", Neutrosophic Sets and Systems, vol. 20, pp. 86-94, 2018.111above
18. Levine, N. (1960), Strong continuity in topological spaces, Amer. Math. Monthly, 67, 269-273.
19. Levine, N. (1963), Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly , 70, 36-41.
20. Levine, N. (1970), Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19, 89-96.
21. Maki, H. (1986), Generalized Λ-sets and the associated closure operator, Special Issue in Communication of Prof. Kazusada Ikeda's Retirement, 139 - 146.
22. M. A. El-Gayer and A. E. F. El Atik, "Topological models of rough sets and decision making of COVID-19". Complexity, vol.2022, Article ID 2989236, 10 pages, 2022. https://doi.org/10.1155/2022/2989236
23. Noiri, T. (1978), A generalisation of perfect functions, J. Korean Math. Soc., 17, 540-544.

24. Noiri, T. (1980), On δ-continuous functions, J. Korean Math. Soc., 16, 161-166.
25. Raja Mohammad Latif (2014), Characterizations of Delta-Open Sets and Mappings in Topological Spaces, May, 12, 1- 29.
26. Yager, R. R. and Abbasov, A. M., Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, vol. 28, no. 5, pp. 436–452, 2013.
27. Yager, R. R., Pythagorean membership grades in multicriteria decision making, IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 958–965, 2014
28. Zadeh, L. A. (1965), Fuzzy sets, Information and control, 8, 338-353.